

Numerical Results for the Lightest Bound States in $\mathcal{N} = 1$ Supersymmetric SU(3) Yang-Mills Theory

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The physical particles in supersymmetric Yang-Mills (SYM) theory are bound states of gluons and gluinos. We have determined the masses of the lightest bound states in SU(3) $\mathcal{N} = 1$ SYM theory. Our simulations cover a range of different lattice spacings, which for the first time allows an extrapolation to the continuum limit. Our results show the formation of a supermultiplet of bound states, which provides a clear evidence for unbroken supersymmetry.

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Supersymmetry (SUSY) plays a fundamental role in the physics of elementary particles beyond the standard model. The understanding of the nonperturbative phenomena of SUSY theories is important since they might explain the supersymmetry breaking at low energies. Besides the relevance for extensions of the standard model, supersymmetric gauge theories also provide insights into nonperturbative phenomena that also occur in QCD, such as confinement of color charges, at least in certain regimes since supersymmetry constrains the nonperturbative contributions. Nonperturbative numerical methods such as lattice simulations are essential to complement and extend the obtained analytical understanding from SUSY models to theories with less or no supersymmetry.

Supersymmetric extensions of the standard model must include the superpartners of the gluons, the so-called gluinos, which are Majorana fermions transforming under the adjoint (octet) representation of SU(3). The gluino would interact strongly, and the minimal theory describing the interactions between gluons and gluinos is $\mathcal{N} = 1$ supersymmetric SU(3) Yang-Mills theory, abbreviated SU(3) SYM theory. The strong interactions between gluons and gluinos are expected to give rise to bound states organised in supermultiplets degenerate in their masses, if supersymmetry is unbroken. The structure of the

supermultiplets has been theoretically investigated in Refs. [1–3]. The boson-fermion degeneracy is expected to appear at the nonperturbative level and, as a consequence, the singlet mesons and glueballs of QCD-like theories have an exotic fermion superpartner, the gluino-gluon, which is a bound state of a single valence gluino with gluons. These predictions are based on formal considerations since a detailed analysis with nonperturbative methods for the theory at low energies has been missing. Unbroken supersymmetry is usually expected due to a nonvanishing Witten index of the theory. However, in presence of relevant nonholomorphic contributions the general picture might be questionable [4] and an investigation without any previous assumption would be desirable.

SU(3) SYM theory is of a complexity comparable to QCD, and Monte Carlo lattice simulations are an ideal *ab initio* approach to investigate this theory. In particular, a study of the mass gap of the particle spectrum requires numerical simulations. As supersymmetry is explicitly broken by any lattice discretization [5–8], it is a challenging task to show that the bound states masses are consistent with the formation of supermultiplets in the continuum limit. It would open up the possibility of much further reaching numerical investigations of SYM theory and correspond to the first step towards a numerical investigation of supersymmetric QCD and gauge theories with extended supersymmetry, since SYM theory is one sector of these theories. Such a result would also provide evidence for the correctness of the conjectured bound state spectrum and for the absence of an unexpected breaking of supersymmetry by the nonperturbative dynamics.

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In this Letter, we focus on the spectrum of bound states of the $\mathcal{N} = 1$ supersymmetric Yang-Mills theory with gauge group $SU(3)$. In previous projects we have investigated SYM theory with gauge group $SU(2)$ [9–11], which can be considered to be a test case for the more realistic $SU(3)$ SYM theory that contains the gluons of QCD. The gauge group $SU(3)$ brings new physical aspects; for instance, it has complex representations in contrast to $SU(2)$, and other types of bound states are possible. The breaking pattern of the global chiral symmetry group is also quite different from the case of $SU(2)$. In particular, in the region of spontaneously broken symmetry, it is expected that CP -violating phases exist, which are related to each other by discrete Z_3 transformations.

We have presented our first data at a single lattice spacing in Ref. [12] together with some estimates of systematic uncertainties. The present Letter is the first final analysis for the lowest chiral supermultiplets of $SU(3)$ SYM theory with a complete chiral and continuum extrapolation.

In the continuum the (on shell) Lagrangian of $SU(3)$ supersymmetric Yang-Mills theory, containing the gluon fields A_μ and the gluino field λ , is

$$\mathcal{L} = \text{tr} \left[-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} + i\bar{\lambda}\gamma^\mu D_\mu \lambda - m_0 \bar{\lambda}\lambda \right], \quad (1)$$

where $F_{\mu\nu}$ is the non-Abelian field strength and D_μ denotes the gauge covariant derivative in the adjoint representation of $SU(3)$. The gluino mass term with the bare mass parameter m_0 breaks supersymmetry softly. The gauge coupling g is represented in terms of $\beta = (6/g^2)$, and the mass in terms of the hopping parameter $\kappa = [1/2(m_0 + 4)]$.

The technical details of our approach for the numerical simulations of $SU(3)$ SYM theory have been described in our previous publication [12]. We employ the lattice discretization of SYM proposed by Curci and Veneziano [13]. In our approach the bare mass parameter is tuned to the chiral limit determined by the point where the adjoint pion $m_{a-\pi}$ mass vanishes. The basic Wilson action for the gluino is in our case improved by the clover term to reduce the leading order lattice artifacts, see Ref. [12] for further details. We have used the one-loop value for the coefficient c_{sw} [14], leading to a one-loop $\mathcal{O}(a)$ improved lattice action at finite lattice spacings a . As indicated by our first results [12], the perturbative c_{sw} is already sufficient to provide a drastic reduction of lattice artifacts even at quite coarse lattice spacings.

Alternative approaches have been investigated for the simulation of SYM theory [15–18], but so far they did not succeed in the continuum extrapolation of the bound state spectrum.

The complexity and the cost of the numerical lattice simulations for this theory is at least as challenging as in corresponding investigations of QCD. Additionally, there

are more specific challenges for the technical realization of numerical simulations of SYM theory, such as the unavoidable explicit breaking of supersymmetry on the lattice. Therefore, the most important task of our project is to demonstrate that the infrared physics emerging from the numerical simulations is consistent with restoration of supersymmetry in the continuum limit.

A further specific challenge is related to the integration of Majorana fermions, which leads to an additional sign factor in the simulation [12]. This Pfaffian sign has to be considered in a reweighting of the observables.

The scale, i. e., the determination of the lattice spacings in physical units in terms of a common observable, is measured from gluonic observables. We are using chirally extrapolated values of the scale w_0 from the gradient flow [19–21]. The chiral values $w_{0,\chi}$ are obtained at each β by a fit of the data to a second order polynomial in the square of the adjoint pion mass in lattice units $(am_{a-\pi})^2$.

An improvement with respect to our work on $SU(2)$ SYM theory, where we extrapolated the observables first to the chiral limit and in a second step to the continuum limit, is that we now use a combined fit towards the chiral and continuum limit. The chiral continuum values $O_{\chi,\text{cont}}$ of the observable O in units of $w_{0,\chi}$ are determined by

$$O(m_{a-\pi}^2, w_{0,\chi}) = O_{\chi,\text{cont}} + c^{(1)}x + c^{(2)}y + c^{(3)}xy, \quad (2)$$

where $x = (w_{0,\chi}m_{a-\pi})^2$ and $y = (a/w_{0,\chi}\beta^2)$ (linear extrapolation). Due to the one-loop clover improvement of the action, we expect leading lattice artifacts to be of $\mathcal{O}(a/\beta^2)$ for on shell observables, which leads to the dependence on the gauge coupling in y . The $\mathcal{O}(a/\beta^2)$ contribution could, however, be very small since considerable improvements have been observed already with the tuning to the one-loop level. In order to compare both cases, we perform additional fits with the leading lattice artifact term $\mathcal{O}(a^2)$, i. e., $y = (a^2/w_{0,\chi}^2)$ in Eq. (2) (quadratic extrapolation).

The main indication for restoration of supersymmetry in lattice simulations presented in this Letter is the formation of mass degenerate supermultiplets. An alternative indication is given by the supersymmetric Ward identities. The violation of the supersymmetric Ward identities in the chiral limit is an indication of lattice artifacts, since chiral symmetry and supersymmetry should be restored at the same point in the continuum theory, if there is no unexpected supersymmetry breaking. The Ward identities also provide a cross check for the tuning of the bare gluino mass parameter. We have found that the Ward identities are consistent with a restoration of supersymmetry, and the leading lattice artifacts are $\mathcal{O}(a^2)$ as found in Ref. [23]. This analysis will soon appear in a separate publication.

We have performed simulations at a large range of values of the inverse gauge coupling β ranging from $\beta = 5.2$ up to $\beta = 5.8$ to search for an optimal window for the continuum

limit extrapolation. In our previous work [12] we have presented the first results for the particle spectrum of SU(3) SYM theory obtained at a single lattice spacing. We have now investigated the systematic uncertainties regarding the finite size effects, the sampling of topological sectors, and the fluctuations of the Pfaffian sign, and found a parameter range where these effects are under control. Only a subset of the considered β range turned out to be reliable for the determination of the bound states. The coarsest lattice spacings (smallest β values) are too far away from the continuum limit, which makes the extrapolation unreliable. The finest lattice spacings (largest β values) suffer from large finite volume effects and a freezing of the topological fluctuations. According to these criteria, our final selection of β values is 5.4, 5.45, 5.5, and 5.6.

In the current Letter, we present the final results for the lightest particles of SU(3) SYM theory. We are now able to combine several different lattice spacings in an

extrapolation to the continuum limit. In comparison to Ref. [12], we have also improved our determination of the bound states, leading to a clearer signal for the particle masses. These methods have been introduced and tested with the data of SU(2) SYM theory in Ref. [11].

The considered states and corresponding interpolating operators are the scalar meson a - f_0 ($\tilde{O}_{a-f_0} = \bar{\lambda}\lambda$), the pseudoscalar meson a - η' ($\tilde{O}_{a-\eta'} = \bar{\lambda}\gamma_5\lambda$), the scalar (0^{++}) glueball, and the fermionic gluino-gluon state $g\tilde{g}$ ($\tilde{O}_{g\tilde{g}} = \sum_{\mu\nu}\sigma_{\mu\nu}\text{Tr}[F^{\mu\nu}\lambda]$), see Ref. [12]. The scalar glueball and the a - f_0 meson are combined in a common variational basis for the scalar channel. The lightest states are expected to form a chiral supermultiplet, which consists of a scalar, a pseudoscalar, and a fermionic spin 1/2 particle. From our previous investigations we expect a reasonable overlap of both the a - f_0 and the scalar glueball with the lightest scalar state, whereas the lightest pseudoscalar state seems to have

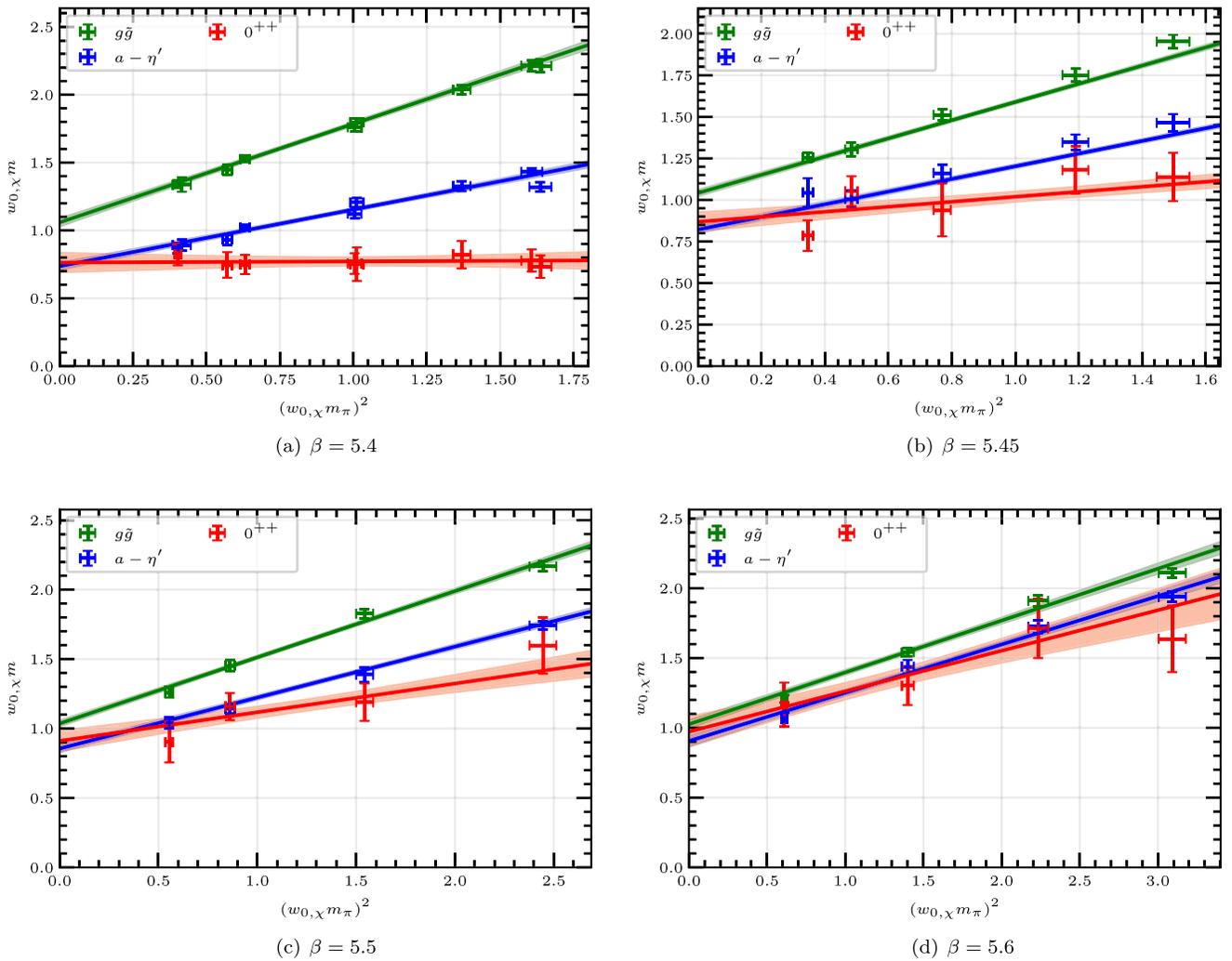


FIG. 1. The chiral extrapolations of the particle masses at the different lattice spacings using the fit function (2) ($y = (a^2/w_{0,\chi}^2)$). The gluino-gluon ($g\tilde{g}$), the pseudoscalar a - η' meson, and the scalar channel (0^{++}), which includes a mixing of the glueball and the a - f_0 meson, are extrapolated to the point where the adjoint pion mass vanishes.

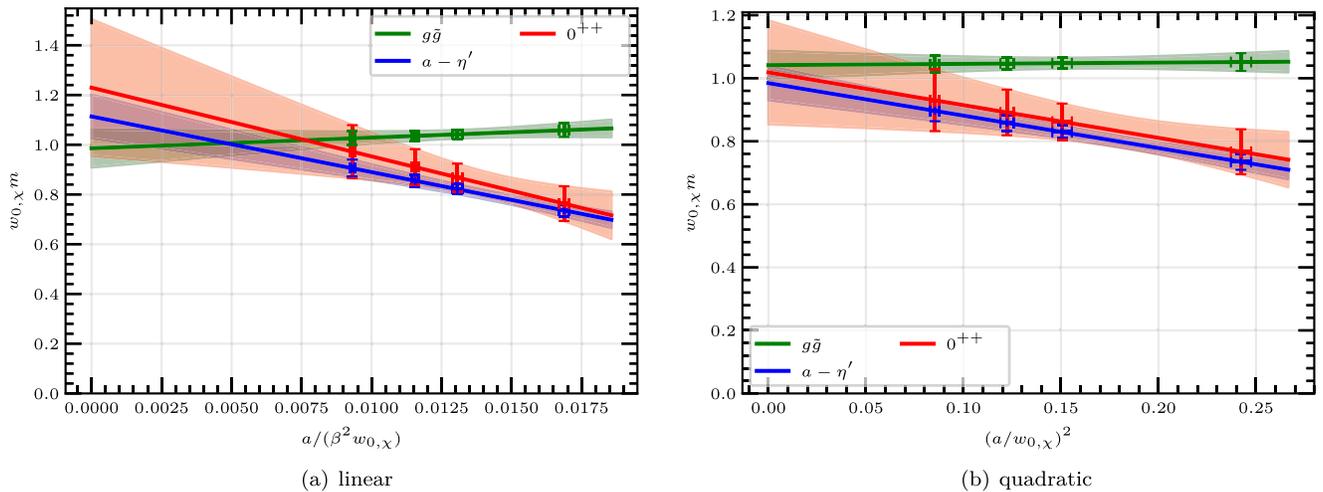


FIG. 2. The extrapolation of the bound states masses to the continuum using the fit function [Eq. (2)] in (a) linear ($y = (a/w_{0,\chi}\beta^2)$) and in (b) quadratic ($y = (a^2/w_{0,\chi}^2\beta^2)$) in the lattice spacing. The data points mark the chirally extrapolated values at simulated lattice spacings. Since the chiral and continuum extrapolation is done simultaneously, they align completely with the fit curves.

a dominant overlap with the $a-\eta'$ rather than with the 0^{++} glueball. Therefore we consider the meson-glueball mixing only in the scalar channel, and neglect, at the moment, the 0^{++} glueball. Note that the measurement of the particle masses in SYM theory is quite challenging, involving only flavor singlet and glueball states.

The chiral extrapolations to the point of vanishing adjoint pion mass $m_{a-\pi}$ are shown in Fig. 1. Away from the chiral point, the particles have different masses and the chiral multiplet splits. This splitting is sizable at least for the coarsest lattice spacings. At these coarsest lattices, the gluino glue becomes the heaviest particle, whereas the scalar particle becomes the lightest state. There is an indication of a remaining mass splitting in the chiral limit at the three coarsest lattice spacings.

At our two finest lattice spacings ($\beta = 5.5$ and $\beta = 5.6$), there is no considerable splitting between the states of the multiplet in the chiral limit. The scalar, pseudoscalar, and fermion masses are degenerate within errors at $\beta = 5.6$ [24]. The 0^{++} state has the largest error of around 20%, and it can not be expected to be more precise than the current glueball measurements in QCD.

A particular problem with our first data at the finest lattice spacing ($\beta = 5.6$) has been the long autocorrelation due to topological freezing. As we have already shown in our previous publication [12], larger volumes allow for more topological fluctuations, but the autocorrelation time of quantities like w_0 is still considerably large.

The three different lattice spacings allow for the first time a complete extrapolation of the lightest states of SU(3) SYM theory to the continuum. Compared to our previous work with an unimproved Wilson fermion action for the investigations of SU(2) SYM theory, the differences of the masses in units of $w_{0,\chi}$ between the different lattice spacings are smaller and the continuum extrapolation is

rather flat thanks to the clover improved fermion action. Due to the weak dependence on the lattice spacing, the linear and quadratic extrapolations are consistent with each other, see Fig. 2. The final results using the two different fit procedures are summarized in the following table [25]:

Fit	$w_0 m_{g\bar{g}}$	$w_0 m_{0^{++}}$	$w_0 m_{a-\eta'}$
Linear fit	0.917(91)	1.15(30)	1.05(10)
Quadratic fit	0.991(55)	0.97(18)	0.950(63)
SU(2) SYM theory in Ref. [11]	0.93(6)	1.3(2)	0.98(6)

For comparison, we have also added the data from our previous investigations of SU(2) SYM theory to the table.

We have finalized our first continuum extrapolation of the lightest bound states in supersymmetric SU(3) Yang-Mills theory. We have found a formation of a chiral supermultiplet in the continuum limit. In combination with the results from an analysis of the supersymmetric Ward identities, this is a good indication for the absence of supersymmetry breaking by the nonperturbative dynamics of the theory. It also shows that the unavoidable breaking of supersymmetry by the lattice discretization is under control in this nontrivial theory.

This important observation opens the way towards several further investigations of SU(3) SYM theory, in particular concerning the phase transitions and chiral dynamics of the theory. In addition, it is the first step towards investigations of supersymmetric QCD and other supersymmetric gauge theories that can not be accomplished without control of the supersymmetry breaking in the pure gauge sector.

Our investigation is based on the approach proposed in Ref. [13], which means that chiral symmetry is broken in a Wilson discretization. Our data indicate that the symmetries are restored by a tuning of the gluino mass parameter and

the approach can be considerably improved by the clover fermion action.

Our results can be compared to our previous analysis of SU(2) SYM theory, presented in Refs. [10,11]. We find that in units of w_0 the masses of the multiplets are compatible with each other. This indicates only a weak dependence of the multiplet mass on N_c .

One interesting additional aspect for further investigations is the continuum limit of the splitting of the multiplet as a function of the soft supersymmetry breaking. Our current data in Fig. 1 show that the slope of the bound state masses as a function of the gluino mass has a significant dependence on the lattice spacing. Therefore the continuum extrapolations away from the chiral limit are more challenging and we plan further investigations in this direction.

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- [25] See the Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevLett.122.221601> for a complete list of simulation runs and obtained measurements.